**Calculating the Number of Permutations**

Consider this example. Five players A, B, C, D, E are available. A team of 3 is to be chosen and then seated for a team photograph on 3 chairs. In how many ways can the photograph be taken ?

Number of objects available = n = 5

Number of objects to be picked = r = 3

Total number of Permutations = $^5p_3$

= $5 \times 4 \times 3 = 60$

(To calculate Permutations, start with 'n' × (n - 1) × (n - 2) × (n - 3) . . . etc. upto 'r' such terms)

e.g. $^7p_3 = 7\times6\times5$ ← r = 3 ∴ 3 terms

e.g. $^{15}p_4 = 15\times14\times13\times12$ ← r = 4 ∴ 4 terms

e.g. $^9p_2 = 9\times8$ ← r = 2 ∴ 2 terms

e.g. $^6p_6 = 6\times5\times4\times3\times2\times1 = 6!$ [ In general, $^np_n = n!$ ]

**Common Situations Given in Permutations / Combinations Problems**

**1.** When a given element is <u>never selected</u>

This is the simplest of situations. Simply ignore the element that should never be present in the selections / arrangements.

*e.g. : In how many ways can you select a committee of 3 from amongst 6 members A, B, C, D, E, F such that 'D' is never a part of a committee ?*

**Soln :** Since D is never present, ignore D

We now have A, B, C, E, F out of which 3 members have to be chosen

∴ n = 5 ; r = 3

∴ Number of ways = $^5c_3 = \dfrac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$ ways

Similarly, if we had to find Permutations,

n = 5, r = 3

∴ Number of Permutations = $^5p_3 = 5 \times 4 \times 3 = 60$ ways

**2.** When one or more element is <u>always present</u>

*e.g. : In how many ways can you select a committee of 5 members from amongst A, B, C, D, E, F, G, H such that 'B' and 'F' are always present in the committee ?*

**Soln :** We have 8 members available A, B, C, D, E, F, G, H. 5 have to be chosen in all.

Since B and F always have to be chosen, select them in advance.

This leaves A, C, D, E, G, H available. As we have already chosen 2 out of 5 members we now need 3 more.

∴ n = 6, r = 3

∴ Number of ways = $^6c_3 = \dfrac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$ ways

*e.g. : In the above question, all conditions remaining the same, if the committee chosen had to be seated in 5 chairs, in how many ways could this be done ?*

**Soln :** We have already found out in the previous question that given the condition, there are 20 ways of selecting a committee. This means that there are 20 distinct groups possible. Each group comprises of 5 members, who can be arranged among themselves in 5 ! ways.

∴ Total number of arrangements = $20 \times 5! = 20\times120 = 2400$ ways

Q-14. *In how many ways can you arrange PAJERO so that the vowels occupy alternate positions ?*

☐ 1] 36 ☑ 2] 72 ☐ 3] 60 ☐ 4] 120

Soln. : There are 3 distinct vowels [ A, E, O ] and 3 distinct consonants [ P, J, R ] in PAJERO

The 3 vowels can occupy positions 1, 3, 5 or 2, 4, 6 i.e. 2 ways

For each such arrangement, the vowels can in turn be arranged amongst themselves in 3 ! = 6 ways.

And for each such arrangement of vowels, the other 3 consonants can also be arranged amongst themselves in 3 ! = 6 ways

∴ Total number of arrangements = $2 \times 6 \times 6 = 72$

Hence [ 2 ]

Q-15. *I want to call-up a friend. I remember the telephone number but am confused about the 3-digit country code. What I do recall however is that all the digits of the country code are different. At the most, how many calls will I have to make to get through to my friend ?*

☐ 1] 89 ☐ 2] 90 ☑ 3] 720 ☐ 4] 810

Soln. : The first digit of the country code could be any digit from 0 - 9 i.e. 10 ways of selecting the first digit.

[ *Note* : A country code could start with zero ]

Having done this, the second digit could be any one of the remaining 9 digits

[ Since the first digit cannot be repeated ]

Having done this, the third digit could be any of the remaining 8 digits

[ both the first and second digits cannot be repeated ]

∴ Total number of permutations possible = $10 \times 9 \times 8 = 720$

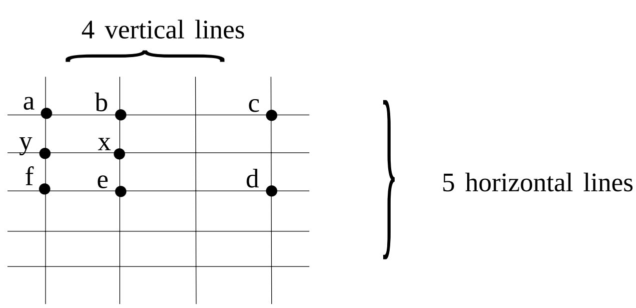
Since one of the 720 permutations has to be right, at most 1 will have to make 720 calls.

Hence [ 3 ]

Q-16. *A set of 5 horizontal lines intersects with another set of 4 vertical lines. How many rectangles are formed?*

☐ 1] 12 ☐ 2] 24 ☑ 3] 60 ☐ 4] 120

Soln. :




From the figure, since the lines are horizontal and vertical, they intersect at 90°.

∴ ☐ abxy is a rectangle. In this way every block which is formed is a rectangle.

In addition to these, we also get a rectangle in the form of 'abef'. Similarly 'acdf' and so on.

In fact, whenever <u>any 2 horizontal lines</u> intersect with <u>any 2 vertical lines</u>, we get a rectangle.

We can select any 2 horizontal lines in $^5c_2$ ways.

We can select any 2 vertical lines in $^4c_2$ ways.

$\therefore$ Number of intersections possible = Number of rectangles

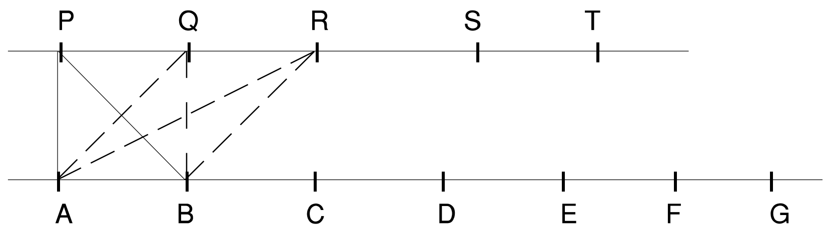
$$= {}^5c_2 \times {}^4c_2 = \frac{5\times4}{2\times1} \times \frac{4\times3}{2\times1} = 60 \text{ ways.}$$

Hence [ 3 ]

Q - 17. *A straight line has 7 points marked on it. Another straight line has 5 more points marked on it. Making use of all these points available, how many triangles is it possible to form ?*

☐ 1] 35 ☑ 2] 175 ☐ 3] 7 ! × 5 ! ☐ 4] 140

Soln. :




In order to form △s, we can choose any <u>pair</u> of points from one of the lines and join them to any <u>one</u> point on the other line

e.g. AB can be joined to P, Q, R, S or T
Similarly, BC can be joined to P, Q, R, S or T
Similarly, AD can be joined to P, Q, R, S or T etc.
Moreover, PQ can be joined to A, B, C, D, E, F or G
QR can be joined to A, B, C, D, E, F or G
QS can be joined to A, B, C, D, E, F or G etc.

From the line containing seven points we can select any 2 points in $7c_2$ ways.

Each of these pairs can be joined to the 5 points on the other line to form △s.

$\therefore$ $^7c_2 \times 5$ ways

Similarly, from the line containing 5 points, we can select any 2 points in $^5c_2$ ways.
Each of these pairs can be joined to any of the 7 points on the other line to form △s.

$\therefore$ $^5c_2 \times 7$ ways

$\therefore$ Total number of △s = $^7c_2 \times 5 + {}^5c_2 \times 7$

$$= \frac{7\times6}{2\times1} \times 5 + \frac{5\times4}{2\times1} \times 7$$

$= 105 + 70$

$= 175$ triangles

Hence [ 2 ]

*NOTE :* It is not necessary that the lines should be parallel.